COMPOSITION OF A PORTFOLIO OF INVESTMENT FUNDS IN THE CASE OF A PERIODIC SUBSCRIPTION

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Abstract: This paper aims to deduce the composition of a portfolio of investment funds when the investor wishes a periodic subscription of the shares belonging to the said portfolio. To do this, starting from the historical series of the selling values and using the model of Roy, we have deduced the profitabilities of each one of the funds in the case of a systematic subscription and so we have calculated the proportion of each fund in the portfolio. These results have been compared using the profitabilities obtained when the investor carries out a simple operation. Finally, this approach can be presented either when the investor can place his contributions according to an arithmetic succession or a geometric one.

Key-words: Investment fund, Roy's model, portfolio, profitability.

INTRODUCTION

The aim of this work is to carry out a mathematical and financial analysis of the profitability obtained by an investor taking into account all the amounts contributed by him because of the periodic subscription of the shares of an investment fund (Cruz Rambaud: 1995, pp. 78-88). In effect, let us consider a person who wishes subscribing, at instant t_0, an investment fund, for which he is going to place a quantity of x_k euros, greater than or equal to another minimum quantity x_{min} $(x_k \, , x_{min})$, a month at instants $t_0, t_1, t_2 \; , t_{n-1}$. Usually, these cash contributions can be carried out either directly, or through a transfer order or through a current account or a savings account in a finance company, that will invest them in the purchase of shares of the investment fund, being C_k the realisable value at t_k of a participation of said mutual fund, $k = 0, 1, 2, \; , n-1$. In the future, we will denote:

1. $C_0, C_1, C_2, \dots, C_{n-1}, C_n$ the purchasing prices of the shares belonging to the investment fund at successive instants of subscription $t_0, t_1, t_2, \; , t_{n-1}$, and the selling value at instant t_n, respectively.
2. n the number of months in which the operation is divided.
3. m the number of years in which the operation is divided, verifying that $m = \frac{n}{12}$.
4. $\frac{x_k}{C_k}$ the number of participations purchased at t_k, $k = 0, 1, 2, \; , n-1$.

So, the equation of financial equivalence at instant tn, using F(t;p) as the valuation financial function, is:

$$\sum_{k=1}^{n} x_k \cdot f(t_k, t_n; p) = C_n\left(\frac{x_0}{C_0} + \frac{x_1}{C_1} + \frac{x_2}{C_2} + \dots + \frac{x_{n-1}}{C_{n-1}}\right), \quad (1)$$

being $f(t_k, t_n; p)$ the financial factor in the interval $[t_k, t_n]$ of the financial function $F(t;p)$ (Gil Peláez, 1992). In the future, we are going to use financial functions belonging to stationary systems and so we will make $tk = k$, $k = 0, 1, \; , n$. A more detailed study of general saving operations through collective investment institutions can be found in Meneu et al. (1994, pp. 225-257).

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This paper is organized in the following manner: Section two studies the monthly and annually profitability of an short-term periodic investment in a mutual fund, using simple capitalization. In section three this profitability is obtained in the case of a long-term investment including three situations: constant and variable (arithmetic and geometric sequences) contributions. Section four exhibits our approach starting from Roy's model, in Section five an empirical application is presented and, finally, Section 6 summarizes and concludes.

CASE OF A SHORT-TERM OPERATION

It is not the more usual case, because an investment in a mutual fund is subscribed with a long-term idea. In any case, this paragraph will show us the methodology to be followed in order to value this operation using simple capitalization. In effect, we are going to suppose that the monthly cash contribution is constant inside the same year. In this case, the outline of the financial operation would be:

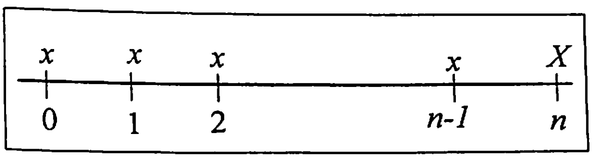

Figure 1: *Case of a short-term operation.*

X being the total amount reimbursed at t_n. In order to calculate the profitability of this operation, we would use the simple capitalization function, establishing the financial equivalence at the final instant and taking into account that usually the financial company would receive a percentage t of the total reimbursed amount:

$$x\cdot(1+n\,i_{12})+x\cdot(1+(n-1)\cdot i_{12})+\cdots+x\cdot(1+i_{12})=(1-t)\cdot X=$$

$$=(1-t)\cdot C_n\cdot\left(\frac{x}{C_0}+\frac{x}{C_1}+\dots+\frac{x}{C_{n-1}}\right).$$

$$n+[1+2+\cdots+(n-1)+n]\,i_{12}=(1-t)\cdot C_n\cdot\left(\frac{1}{C_0}+\frac{1}{C_1}+\cdots+\frac{1}{C_{n-1}}\right),$$

from which it is deduced that:

$$i_a=\frac{2\cdot\left(\frac{(1-t)\cdot C_n}{H_n}-1\right)}{n+1}, \qquad (2)$$

H_n being harmonic average of $C_0, C_1, \ , C_{n-1}$.

Finally:

$$i = 12\, i_{12}. \qquad (3)$$

In this case, the profitability is the mean of the profitabilities of each acquisition value weighted by the time of permanence in the portfolio of the investor. In effect:

$$i_{12}=\frac{\frac{\frac{(1-t)\cdot C_n}{C_0}-1}{n}\cdot n+\frac{\frac{(1-t)\cdot C_n}{C_1}-1}{n-1}\cdot(n-1)+\cdots+\frac{\frac{(1-t)\cdot C_n}{C_{n-1}}-1}{1}\cdot 1}{\frac{n\cdot(n+1)}{2}}=$$

$$=\frac{(1-t)\cdot C_n\cdot\left(\frac{1}{C_0}+\frac{1}{C_1}+\cdots+\frac{1}{C_{n-1}}\right)-n}{\frac{n\cdot(n+1)}{2}}=$$

$$=\frac{(1-t)\cdot C_n\cdot\frac{n}{H_n}-n}{\frac{n\cdot(n+1)}{2}}=\frac{2\cdot\left(\frac{(1-t)\cdot C_n}{H_n}-1\right)}{n+1}.$$

CASE OF A LONG-TERM OPERATION

As we have indicated previously, it is the more usual case (retirement, studies, etc.). Moreover, we can enumerate three sub-cases inside this one (Cruz Rambaud, 1998: 50-58):

Constant Contributions

The outline of this operation would be the same as the former paragraph, but taking into account that now we would use the compound capitalization in order to find the profitability:

$$x\cdot(1+i_{12})^{n}+x\cdot(1+i_{12})^{n-1}+\cdots+x\cdot(1+i_{12})=C_n\cdot\left(\frac{x}{C_0}+\frac{x}{C_1}+\cdots+\frac{x}{C_{n-1}}\right),$$

$$x\cdot\ddot{S}_{\overline{n}|i_{12}}=x\cdot\frac{n\cdot C_n}{H_n}\Rightarrow\ddot{S}_{\overline{n}|i_{12}}=\frac{n\cdot C_n}{H_n}\qquad(4)$$

from which the value of i_{12} is deduced, by means of the use of financial tables or a spreadsheet. Alternatively:

$$12\cdot x\cdot\ddot{S}^{(12)}_{\overline{m}|i}=x\cdot\frac{n\cdot C_n}{H_n}\Rightarrow\ddot{S}^{(12)}_{\overline{m}|i}=\frac{n\cdot C_n}{12\cdot H_n}\qquad(5)$$

from which the value of i is deduced.

Increasing Contributions Variable According to an Arithmetic Sequence

Let us suppose, to simplify, that the duration of the operation is an exact number of years and that the monthly contribution annually increases according to an arithmetic sequence with common difference d:

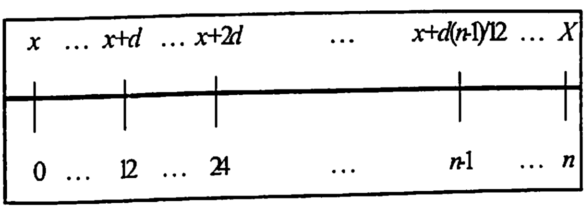

Figure 2: *Case of a long-term operation. Increasing contributions according to an arithmetic sequence.*

$$\ddot{S}(12x,12d)^{(12)}_{\overline{m}|i}=X=C_n\cdot\left[\frac{x}{C_0}+\frac{x}{C_1}+\cdots+\frac{x}{C_{11}}+\right.$$

$$+\frac{x+d}{C_{12}}+\frac{x+d}{C_{13}}+\cdots+\frac{x+d}{C_{23}}+\frac{x+2d}{C_{24}}+\frac{x+2d}{C_{25}}+\cdots+\frac{x+2d}{C_{35}}+$$

$$\left.+\cdots+\frac{x+(m-1)d}{C_{n-12}}+\frac{x+(m-1)d}{C_{n-11}}+\cdots+\frac{x+(m-1)d}{C_{n-1}}\right].$$

By applying the distributive property and re-arranging the parentheses, it remains:

$$C_n\cdot\left[x\left(\frac{1}{C_0}+\cdots+\frac{1}{C_{n-1}}\right)+d\cdot\left(\frac{1}{C_{12}}+\cdots+\frac{1}{C_{n-1}}\right)+\right.$$

$$\left.+d\cdot\left(\frac{1}{C_{24}}+\cdots+\frac{1}{C_{n-1}}\right)+\cdots+d\cdot\left(\frac{1}{C_{n-12}}+\cdots+\frac{1}{C_{n-1}}\right)\right]=$$

$$=C_n\cdot\left[x\cdot\frac{n}{H'_n}+d\cdot\left(\frac{n-12}{H'_{n-12}}+\frac{n-24}{H'_{n-24}}+\cdots+\frac{12}{H'_{12}}\right)\right],$$

From which it is deduced that:

$$\ddot{S}(12x,12d)^{(12)}_{\overline{m}|i}=C_n\cdot\left[x\cdot\frac{n}{H'_n}+d\cdot\sum_{k=1}^{m-1}\frac{12k}{H'_{12k}}\right],\qquad(6)$$

H'_{12k} being the harmonic average of the $12k$ last selling values of the investment fund. Now then, an alternative development of the last expression:

$$C_n\cdot\left[x\cdot\left(\frac{1}{C_0}+\cdots+\frac{1}{C_{n-1}}\right)+d\cdot\left(\frac{1}{C_0}+\cdots+\frac{1}{C_{n-1}}\right)-d\left(\frac{1}{C_0}+\cdots+\frac{1}{C_{11}}\right)+\right.$$

$$+d\cdot\left(\frac{1}{C_0}+\cdots+\frac{1}{C_{n-1}}\right)-d\cdot\left(\frac{1}{C_0}+\cdots+\frac{1}{C_{23}}\right)+\cdots$$

$$\left.\cdots+d\cdot\left(\frac{1}{C_0}+\cdots+\frac{1}{C_{n-1}}\right)-d\cdot\left(\frac{1}{C_0}+\cdots+\frac{1}{C_{n-13}}\right)\right]=$$

$$=C_n\cdot\left[\frac{n}{H_n}[x+(m-1)d]-d\cdot\left(\frac{12}{H_{12}}+\frac{24}{H_{24}}+\cdots+\frac{n-12}{H_{n-12}}\right)\right]=$$

$$=C_n\cdot\left[\frac{n}{H_n}\cdot(x+md)-d\cdot\left(\frac{12}{H_{12}}+\frac{24}{H_{24}}+\cdots+\frac{n-12}{H_{n-12}}+\frac{n}{H_n}\right)\right],$$

would lead us to the following equality:

$$\ddot{S}(12x,12d)^{(12)}_{\overline{m}|i}=C_n\cdot\left[\frac{n}{H_n}(x+md)-d\sum_{k=1}^{m}\frac{12k}{H_{12k}}\right],\qquad(7)$$

H_{12k} being the harmonic average of the $12k$ last selling values of the investment fund.

Table 1: *Estimated Values For Mean And Variance*

	PERT ESTIMATES	
FUND	MEAN	SAMPLE VARIANCE
FONDTESORO	0.026297274	1.16551E-05
BK DINERO 98	0.03151959	1.79567E-05
CITICASH FUND	0.031576958	1.49335E-05
CAM DINERO PREMIER	0.029962225	1.542E-05
FONDO VALENCIA ORO	0.028492836	1.46152E-05
BESTINVER MIXTO	0.106938986	0.00573264
EUROAGENTES PREVISION	0.116249883	0.005308738
FIDEFONDO	-0.008630831	0.000392384
BBVA GARANTIA 2	0.009337543	0.002124771
ALCALA BOLSA	0.032666717	0.003173678
BESTINFOND	0.134691321	0.012820506
BESTINVERBOLSA	0.139600515	0.013974051
CHASE ACCIONES EUROPEAS	-0.146679434	0.01995762
CHASE UTILITIES	-0.013126445	0.009251388
MUTUAL FONDO VALORES	-0.12697328	0.021384687

Table 2. *Portfolio Composition for a Reservation Return Of 1% in the Case of Periodic Suscription.*

FUND	COMPOSITION
CUENTA FONDTESORO, FIAMM	1%
BK DINERO 98, FIAMM	21%
CITICASH FUND, FIAMM	45%
CAM DINERO PREMIER FIAMM	29%
FONDO VALENCIA ORO, FIAMM	0%
BESTINVER MIXTO, FIM	0%
EUROAGENTES PREVISION, FIM	2%
FIDEFONDO, FIM	0%
BBVA GARANTIA 2	0%
ALCALA BOLSA, FIM	0%
BESTINFOND	0%
BESTINVERBOLSA	0%
CHASE ACCIONES EUROPEAS	0%
CHASE UTILITIES	0%
MUTUAL FONDO VALORES	1%

Estimated Return: 3.6%; Expected Risk: 3.65%

Table 3: *Portfolio Composition For a Reservation Return of 1% in the Case of a Simple Operation.*

FUND	COMPOSITION
CUENTA FONDTESORO, FIAMM	0%
BK DINERO 98, FIAMM	15%
CITICASH FUND, FIAMM	56%
CAM DINERO PREMIER FIAMM	24%
FONDO VALENCIA ORO, FIAMM	3%
BESTINVER MIXTO, FIM	0%
EUROAGENTES PREVISION, FIM	0%
FIDEFONDO, FIM	0%
BBVA GARANTIA 2	0%
ALCALA BOLSA, FIM	0%
BESTINFOND	0%
BESTINVERBOLSA	0%
CHASE ACCIONES EUROPEAS	0%
CHASE UTILITIES	0%
MUTUAL FONDO VALORES	1%

Estimated Return: 3.8%.
Expected Risk: 0.54%.

As we can observe, the results in the case of a simple operation are quite similar, but better than in the case of periodic subscription. This is logical, because the money is invested more time, and the tendency of the market is higher in the FIAMMs, that represent all the portfolio.

Table 4. *Portfolio Composition for a Reservation Return of 0% in the Case of Periodic Suscription Without Fiamms.*

FUND	COMPOSITION
BESTINVER MIXTO, FIM	18%
EUROAGENTES PREVISION, FIM	28%
FIDEFONDO, FIM	10%
BBVA GARANTIA 2	0%
ALCALA BOLSA, FIM	30%
BESTINFOND	8%
BESTINVERBOLSA	6%
CHASE ACCIONES EUROPEAS	0%
CHASE UTILITIES	0%
MUTUAL FONDO VALORES	0%

Estimated Return: 7.98%.
Expected Risk: 54%.

In the last application, it is shown how, for a short increase of the estimated return, a quite higher increase of the expected risk is obtained. This result is according with the evolution of the markets and the logical of finances.

CONCLUSION

Starting from the historical series of the selling values of fifteen investment funds, we have deduced the composition of a portfolio in order to maximize the profitability for a given level of risk. More specifically, we have chosen the realisable values of the five better FIAMM, the five better fixed interest funds and the five better variable interest funds, from January 2001 to July 2002 in the Spanish Stocks Market. With these data and taking as reference date the selling value of each mutual fund, we have calculated the profitabilities in two cases:

- When the operation is simple (that is to say, one deposit and one withdrawal of capital):

$$i_k = \frac{P_k - P_0}{P_0}\frac{12}{k}$$

  being i_k the profitability corresponding to *k*-th month, *P*k the selling value of a share at the end of the *k*-th month and *P*0 the realisable value at the end of the first month.

  Then $i = 12 \cdot i_k$

- When the operation is composite (that is to say, several deposits and one withdrawal of capital) and the cash contribution is constant. In this case, we must use the formulae of sections 2 and 3.

In both cases, and using the statistical distribution of the classical PERT (Suárez Suárez, 1995), we have estimated the mean and the variance and so deduced the proportion of each participation in the portfolio of the investor. The general approach when the contribution of the investor increases according to an arithmetic or a geometric sequence is presented but the empirical application is left to the reader.

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