

OPTIMISATION OF INVESTMENTS DECISIONS

Is the underdeveloped world skipping the opportunity?

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Abstract

One of the basic principles of solving problems is separating the problem-solving phase from implementation. It is believed, in most cases, that if a correct algorithm is pursued in the problem-solving phase, then implementation is likely to lead to a desired solution. Sometimes, however, the problem-solving phase is carried out, and probably simulated, in an environment quite distinct from the one in which implementation eventually takes place. Even with a mathematically sound model, and an acceptable logical-flow of events, what may be looked at as side-issues can seriously hamper implementation.

In this paper, we'll be looking at the concept of investment-decision making, and the efforts that are constantly made to ensure that correct decisions are reached. We'll be arguing that investment decisions optimization models are not necessarily a panacea to poor investment outcomes in the least developed world, albeit being rigorously efficient. We devote more time to the problem-solving phase, and as a result we postpone implementation, without calling it off altogether.

Introduction

Our primary objective, in this paper, is to present a reasonably self-contained discussion of Investment Decisions Analysis based on two normative models, under different conditions. Investment Decisions Analysis is the driving force behind any economic prosperity, and a retarding factor behind any economic failure. Over the period of the past thirty years or so, a good number of Tanzanian parastatals rose and fell. The past decade or so has witnessed an increasing number of them going bust. Something should be seriously wrong, and a search for a solution has to be mounted. And what do we come up with? A nice set of two optimization models, Dynamic Programming and Contingent Claims Analysis.

At different times over the past four decades, Dynamic Programming, and more recently Contingent Claims Analysis have been given a great role to play in Investment Decision-making. The basic reason behind this development has been the need to come up with

the best decision out of a number of possible alternative decisions. Naively said, there has been a need to isolate the best from the rest. Most models in finance are statistically based, and as such they do represent a relationship without indicating any course of action to be taken. These include the Market Model, the Capital Asset Pricing Model, etc.

Apparently, neither the Market Model nor the CAPM does indicate what particular values of beta and sigma respectively, should be adopted. Such models are said to be descriptive. On the other hand, normative models may contain descriptive submodels, but differ from the former in that with them, it is possible to determine the best choice, or course of action, usually referred to as the optimal choice. It is in the above context that we'll be conducting our discussion. We start off with a brief theoretical exposition of the two models, the underlying assumptions and finally we proceed to issues relating to their applicability in both time and space.

PREVIEW OF THE MODELS

The two methods are, in fact, very similar and would usually lead to identical results in many applications. But before we proceed further, we would like to make a distinctive remark about the nature of these methods. Dynamic Programming is a mathematical programming tool generally applied in dynamic optimization. Its power derives from its ability to break-up a single large problem into a sequence of small problems, hence making it more manageable. Dynamic Programming is an approach to solving a single large problem by solving a sequence of small problems. As such it allows us to solve a time-dependent problem as a sequence of one-period problems, where the parameters of each period depend on the period being considered.

On the other hand, Contingent Claims Analysis derives from financial economics. It can easily be understood by considering an investment project that is characterized by a stream of cash in-flows and out-flows that are uncertain through time due to being a function of some uncertain events. If you own such a project, or profits from it, then you own a valuable asset. Such assets are widely traded, therefore your asset should've a market value. Even if the asset you own is not directly traded, a portfolio of traded assets should be able to replicate the pattern of returns from your asset. In this case, the value of your asset must be equal to the value of the replicating portfolio, otherwise room will be created for arbitrage opportunities. It's only when you know the value of your asset that you can determine the optimal investment policy.

CONTINGENT CLAIMS ANALYSIS: An Introductory Look

Contingent Claims Analysis can basically be discussed from three perspectives, viz; the option to wait, abandon, and temporarily suspend a project. Our exposition here will be much reflective of the theory of Contingent Claims Analysis as expounded by Dixit and Pindyck¹ (1994). With temporary suspension and abandonment, the initial assumption of irreversibility, which is associated with the conventional NPV rule is relaxed, thus providing

escape routes as the operating profit of a project persistently goes negative. To quote Dixit and Pindyck, "When we consider investment and abandonment(or entry and exit) together, a firm's optimal decision is characterized by two thresholds. A sufficiently high current level of profit, corresponding to an above-normal rate of return on the sunk cost, justifies investment or entry, while a sufficiently large level of current loss leads to abandonment or exit. Now suppose the current level of profit is somewhere between these two thresholds. Will we see an active firm?"² The answer is provided, not by the current state of the underlying stochastic variable, but by the historical profit fluctuations.

This approach looks at Investment from a broader perspective than simply the act of incurring an immediate cost in anticipation of future rewards. From this perspective, a firm that closes down a loss-making business is also investing, and so will be a portfolio manager who postpones investment in anticipation of higher rewards later. Investment is considered to be either fully or partially irreversible, in the sense that no initial outlay can fully be salvaged should things go wrong. Investment is also characterized by uncertainty, which surrounds both inputs and outputs. Finally, investment can be postponed or delayed in anticipation of more useful information. Optimal decisions are reached through a continuous interaction of the three characteristics. Failing to recognize this important interaction, spells a tremendous failure in identifying, modeling and solving financial investment problems.

A number of neoclassical investment rules are known. Among them, the conventional NPV, IRR rules and what is known as Tobin's q , developed by Tobin in 1969. In a typical class-room type statement, the Net Present Value rule states that "...accept an investment if its NPV is positive, otherwise do not...". The NPV itself is computed as the sum of an outlay and all anticipated discounted cash flows of the investment. A very crucial item in the computation of the NPV is the determination of "what to discount". Brealey & Myers³ suggest

that three general rules be applied in determining what to discount, i.e. considering only cash flows, considering them only on an incremental basis, and being consistent with the treatment of inflation.

The Internal Rate of Return investment criterion derives from the NPV rule. It can simply be stated to be the highest interest rate we can afford to pay out and still break even. This is therefore the discount rate which make the NPV equal to zero. Computing the IRR requires a simple trial and error approach, playing around with the rate until you come up with the one that equates the NPV to zero. The use of this criterion will therefore stipulate that the discount rate not exceed a given level, for the project to be accepted.

Tobin's q is simply the ratio of the market value of assets to estimated replacement cost. This criterion was developed by J. Tobin⁵ almost thirty years ago. The ratio looks like the market-to-book ratio, but there are a number of differences. The market value of assets in Tobin's q ratio combines all the firm's debt and equity securities, while the denominator puts together all the firm's assets, entered at what it would cost to replace them. According to Tobin, firms have an incentive to invest when q is greater than one. This is when capital equipment is worth more than the cost of replacing it. The incentive to invest vanishes as capital equipment falls below the cost of replacing it. In this case, it'll be cheaper to acquire assets through merger than procuring new assets.

The foregoing rules have been applied, almost conventionally, both in business school classes as well as in industry. However, experience over the past few years has shown that all investment undertakings are associated with opportunity costs, and that these opportunity costs can be too large to ignore. More concern has been directed towards interest rate dynamics, even though interest rates have very little effect on investment demand. Recent studies have shown that the sensitivity of these opportunity costs to uncertainty over the future value of the project is

probably higher than that of interest rates⁶. Some have argued that this spells the failure of the neoclassical investment theory in providing reliable empirical models of investment behaviour.

The intuitive understanding of real options can be drawn from practical experience. While the neoclassical investment theory gives a clear cut decision to invest when a given condition is met, for instance invest if the NPV is positive, this is usually not always the case. Dixit and Pindyck have the following to say: "Firms invest in projects that are expected to yield a return in excess of a required, or 'hurdle', rate. ...such hurdle rates are typically three or four times the cost of capital. ...firms do not invest until price rises substantially above long-run average cost. ...and price can fall substantially below average variable cost without inducing disinvestment or exit"⁷. Why so? It all seems to conflict with the neoclassical investment theory! This puzzle can be resolved once irreversibility and option value are accounted for.

DELAYING INVESTMENTS AND IRREVERSIBILITY

We started off by making a statement about the assumptions with which the conventional NPV rule comes with, among them was the assumption that investment is fully reversible. Contingent Claims Analysis comes with an assumption that investment is either fully or partially irreversible. On the other hand there's the possibilities of delaying investments should be looked at with care. Delaying an investment could mean risking entry by competitors, stated in other words, quick investment is probably good as it may preempt investment by competitors, it could also mean getting there first. All these costs, however, must be weighed against potential benefits associated with the arrival of new information, which are often enormous. Stated in other words, the value of waiting must always be traded off against the loss of the profit flow incurred during the period in which investment was postponed. The optimal decision made is, as a rule, a function of the model's underlying risk and the discount rate.

In terms of irreversibility, investment opportunity is similar to a financial call option in that a financial call option gives the holder the right, not obligation, to buy an asset. However, exercising the option is irreversible, since if the option is out of the money and the holder decides to simply walk away s/he can not retrieve the option price that was paid to acquire the option. The option to invest is therefore very similar to the foregoing case. If a firm acquires an investment opportunity in anticipation of a commensurate return, and at the end of the day, things go the other way, it can only lose what it spent to obtain the investment opportunity, since it invests only if the price has gone up, but not if it has gone down.

The applicability of Contingent Claims Analysis comes with the assumption that the risk of the investment can be replicated, or at least spanned by a portfolio of existing assets. This assumption is by all standards too much demanding.

DYNAMIC PROGRAMMING: An Introductory Look

So what, if the basic assumption associated with Contingent Claims Analysis is much too demanding? There's an immediate alternative to Contingent Claims Analysis, and that is Dynamic Programming. Optimization problems are usually associated with a number of different factors. The possible identification of those factors and their relationships, and the functional formulation of the underlying problem, together make the problem a subject of mathematical programming.

DYNAMIC PROGRAMMING

Dynamic Programming is a branch of mathematical programming. The method was first formalized in a book by Richard Bellman⁸, who is considered the father of Dynamic Programming. With it, a wide range of extreme problems can be attempted. Most of such problems, however, are of a dynamic nature, and as such in solving them time as a variable or the sequencing of operations should be considered.

There are four basic assumptions underlying the method of Dynamic Programming⁹. The primary assumption is the segmentation of a single large problem into manageable sub-problems. The other three assumptions are: The single large problem can be solved through sequential decisions made at different stages, sub-problems are more manageable than a single large one, and the Bellman's Principle of Optimality, which states that: "An optimal set of decisions (a policy) has the property that if a given decision is optimal, then all subsequent decisions depending on the decision must also be optimal."

The principle of optimality together with the segmentation of a single large problem into a sequence of small problems, suggests the backward approach used in Dynamic Programming problem solving methodology. If we do so, argues Bellman, we are assured of finding the optimal set of decisions.

Essentially, Dynamic Programming looks at decision-making in multistage processes. In general, Dynamic Programming seeks to find a solution to an optimization problem in a sequential manner. Unlike other mathematical programming methods like linear, integer, or quadratic, it is not a single-solution algorithm, but an approach to solving a single large problem by solving a sequence of small problems. As such it allows us to solve a time-dependent problem as a sequence of one-period problems, where the parameters of each period depend on the period being considered.

TERMINATION OR CONTINUATION DECISIONS

One of the great Socrates' tenets was that a man's happiness or well-being depended directly on the badness or goodness of his soul, and that no one ever wishes for anything but true good. In this sense then, any wrong-doing can only be considered involuntary. We'll base our argument on a simplified case in which an investor has a binary choice. We'll be basing our reasoning on the decisions which the Tanzanian¹⁰ government had to make concerning the ill-fated parastatal organizations over the past few years. We

therefore take-off with a belief that an investor wishing for nothing other than true good, will continue with the project only if it's worth doing so, otherwise s/he will sell it off.

Now let's seek for the true good by going by the following illustration. Consider a two period investment opportunity to costlessly produce the product X . The investment is completely irreversible, and the initial outlay is I . Only a single unit of X is to be produced in perpetuity. The prevailing interest rate is r . Now suppose the prevailing price is P_0 , and in the next period, i.e. period one, it will either go up by the magnitude u , with the probability $p(u)$, or go down with the magnitude d , with the probability $1 - p(u)$. In those two cases, the new levels of price will be $(P_0 + P_0 u)$ and $(P_0 - P_0 d)$ respectively.

Now let's assume that the investment opportunity is a now-or-never choice. So let's denote the expected present value of the revenues accruing from the investment by V_0 . Notice that this value will be the sum of the current price of the asset and discounted expected price of the asset. Also notice that discounting starts in year one to infinity, and the expected price is computed in the normal way by simply weighting the expected price movements by their probabilities of occurrence. Thus we get:

$$(1) \quad V_0 = P_0 + [p(u)P_0(1+u) + (1-p(u))(1-d)P_0] \left[\frac{1}{(1+r)} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3} + \dots \right].$$

Notice that the discounting factor is a convergent series, a special case of the geometric progression¹¹.

To get a clear intuition of this series consider the sum of a geometric progression with the first term one and the common ratio a half. This is the sum: $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$. This is a special case of geometric progression, and its sum converges to two. It can immediately be noted that adding

the first and the second terms gives $1\frac{3}{4}$, which is less than two by exactly the last term added,

adding the first, second, and third terms gives $1\frac{7}{8}$, which is less than two by $\frac{1}{8}$, exactly the last

term added, it so goes on to infinity. Said in other words, the sum of the series gets closer and closer to two, but never actually reaches it. It can be made as close to two as possible by taking a sufficiently large number of terms.

The value two, is called the limit of the sum of this series. Series having such a limit are said to be convergent¹². Generally, the sum converges to a number given by the first term divided by one minus the common ratio. Symbolically, and by going straight to our discounting factor, where both

the first term and the common ratio are given by $\frac{1}{(1+r)}$, the sum will converge at:

$$(2) S_{\infty} = \frac{\frac{1}{(1+r)}}{1 - \frac{1}{(1+r)}} = \frac{\left(\frac{1}{(1+r)}\right) (1+r)}{1+r-1} = \frac{1}{r}.$$

Indeed, our expected PV of the revenues accruing from the investment becomes:

$$(3) V_0 = P_0 + \left[p(u)(1+u)P_0 + (1-p(u))(1-d)P_0 \right] \frac{1}{r}. \text{ Expanding the coefficients of } \frac{1}{r}$$

yields:

$$V_0 = P_0 + \left[(p(u) + p(u)u)P_0 + (1-p(u)-d + p(u)d)P_0 \right] \frac{1}{r} \quad \downarrow$$

$$\downarrow V_0 = P_0 + \left\{ P_0 [1 + p(u)(u+d) - d] \right\} \frac{1}{r} \quad \downarrow$$

$$\downarrow rV_0 = rP_0 + P_0 [1 + p(u)(u+d) - d] \Rightarrow rV_0 = P_0 [r + (1 + p(u)(u+d) - d)]$$

Which eventually becomes:

$$(4) rV_0 = P_0 [r + 1 + p(u)(u+d) - d] \Rightarrow V_0 = P_0 [1 + r + p(u)(u+d) - d] \frac{1}{r}.$$

The investment is made if there's a positive payoff, i.e. if the value of the investment is greater than the amount to be invested. In this case therefore the investing firm or individual realizes the difference $V_0 - I$. If, on the other hand, $V_0 < I$, the investment opportunity is skipped, and the investor gets

zero. The break even point, $V_0 = I$, offers zero rewards whether investment is made or not. So if we denote the net payoff from investing now by Ω_0 , then the same is given as:

$$\Omega_0 = \max [V_0 - I, 0].$$

Now let's take a turn and look at the possibility of postponing the investment to period one, and thereafter conditions remain the same. We said above that with a probability $p(u)$, the price could go up by u , and with a probability $1 - p(u)$, drop by d . Therefore if the firm decides to wait, the price in period one will be:

$$(5) P_1 = \begin{cases} P_0 + P_0 u & \text{with probability } p(u), \\ P_0 - P_0 d & \text{with probability } 1 - p(u). \end{cases}$$

As in the previous illustration, this stream of cashflows can be discounted back to period one. Notice that though the price at period one can be computed as the expected price in that period by weighting the jumps by the appropriate probabilities, we prefer to adopt a generalized symbol, P_1 , instead. Thus we can express the investment value as:

$$(6) V_1 = P_1 + \frac{P_1}{(1+r)} + \frac{P_1}{(1+r)^2} + \frac{P_1}{(1+r)^3} + \dots$$

Here again, we've P_1 as the first term, and

the common ratio $\frac{1}{(1+r)}$. The value of investment is therefore given as:

$$(7) V_1 = \frac{P_1}{1 - \frac{1}{(1+r)}} = \frac{P_1(1+r)}{1+r-1} = \frac{P_1(1+r)}{r}.$$

And here again, investment will be made

only if there's a positive net payoff F_1 , thus $F_1 = \max[V_1 - I, 0]$. Notice that both V_1 and F_1 are random variables. Again, notice that the expected net payoff in period one can be computed basing on the currently available information as follows:

$$(8) E_0[F_1] = p(u) \max \left[(1+u) \frac{P_0(1+r)}{r} - I, 0 \right] + (1-p(u)) \max \left[(1-d) \frac{P_0(1+r)}{r} - I, 0 \right].$$

Equation (8) is known as the continuation value and we can verbally express it in the following way: The value of the investment in period one depends on whether the price goes up or goes down, and they both occur with given probabilities. Therefore weighting the expected price levels by their corresponding probabilities and summing up, gives, in the very ordinary way, the expected value of investment as above.

Now notice that if the decision is made now, there are only two choices available to the investor, that's either invest now, if $V_0 - I > 0$, and pocket the difference, or wait and invest in period one and pocket the continuation value given above. The optimal decision is the one that makes the investor pocket more. Notice again, that since the continuation value starts in period one, to be brought to comparable terms with $V_0 - I > 0$, it must be discounted back to the current period. And finally, if we denote the NPV of the investment opportunity by F_0 , we get the following expression:

$$(9) F_0 = \max \left\{ V_0 - I, \frac{1}{(1+r)} E_0(F_1) \right\}.$$

Equation (9) reflects the basic idea of Dynamic Programming, precisely, it represents a special case of the general Bellman equation¹³. In this instance we had only two periods, but even with a multiple number of periods, the same procedure can recursively be applied. It should also be noted that the

now-or-never choice we had earlier, is a rigid one, and the choice to wait and see if anything useful unfolds, is a flexible one. Any difference between the two net payoffs, $F_0 - \Omega_0$, can only reflect the value of the extra flexibility, that's the value of the option to wait. Then a crucial question arises: Why should this flexibility have value?

To answer the above question let's look at what happens when an investor decides to wait. First of all, by waiting, the revenue in the current period P_0 , is foregone, this seems to suggest that investment should be made immediately. But waiting also implies not dishing out the amount I , probably a good decision if the same amount can, in the meantime, earn positive interest. But notice that the future trend of investment costs plays an important role here. If they are expected to rise, it may probably be reasonable to invest now, but if they are expected to go down, waiting may be preferred.

TERMINAL PAYOFF V/S CONTINUATION VALUE

Let's assume that at a given point in time T , the decision process is forced to terminate with the final payoff being a function of the state reached, and denoted by $\Omega_T(x_T)$. In other words we either have to stop the process and take the termination payoff, or wait a period more, i.e. continue for one period, when a similar choice arises. So by terminating or stopping we refer to making the investment now, and by continuing we mean waiting. For an illustration consider a poorly performing firm that is contemplating closure. The firm has basically two options, to either wait one more period, that's continue and realize either a loss or, if lucky, profit, or close down and realize some scrap value of its assets minus any relevant costs.

Now if we denote the profit by $Pr(x)$, and the termination payoff by $\Omega(x)$, then equation (9), or rather the Bellman equation becomes:

$$(10) F(x) = \max \left\{ \Omega(x), Pr(x) + \frac{1}{(1+\rho)} E[F(x')|x] \right\}$$

Now looking at equation (10) above we immediately realize that for it to be maximized, as the RHS stipulates, there should be different values of the state variable x . Apparently, it's not likely to have the same values of the state variable equating the net payoff from termination to those from continuation. Drawing a line of demarcation between those values of the state variable which maximize the function through termination and those which do the same through continuation can be a truly challenging task. We find this to be a crucial issue especially in those countries with severely constrained investment capital, rudimentary forecasting tools, non-positive real interest rates, and generally poor economic progress. In most such cases the decision to either terminate or continue is not optimal-oriented.

A number of authorities seem to suggest the use of the so-called single cut-off point, which we can denote by x^* , with termination optimal on one side and continuation on the other. However, as we shall soon see, such an approach is quite impractical in our parts of the world. Given the prevailing capital problems in the underdeveloped world, there'll be an option to incur more and more losses now, tomorrow, the day after, etc. in the hope of a turnaround in an unforeseeable future¹⁴. In other words, irrationally forcing matters well beyond any rational cut-off point, which usually leads to financial stress.

It should be noted that Dynamic Programming and Contingent Claims Analysis would usually lead to the same results in many different applications. The illustration made above, based on the former model can be replicated with the latter, the only exception being that the discount rate will then be an endogenized quantity. The two models differ only in the underlying assumptions. In this paper we argue that neither of them is a direct panacea to investment problems of the third world. The

following section addresses just that.

WHAT THE MODELS CAN NOT TELL

Having explored the power of the models and the optimal results that they yield when all the necessary conditions are met, it's time now to turn to the question relating to the robustness of the models. It is important to note that just like any other models, both Dynamic Programming and Contingent Claims Analysis are constructed under given assumptions, conditions and requirements. Precision of their results will depend much on these assumptions holding, conditions met, and requirements fulfilled.

THE METHODS' BASIC ASSUMPTIONS

Both Dynamic Programming and Contingent Claims Analysis, have inherent problems, which should be carefully considered in applying them. The first one involves the problem relating to the choice of the discount rate, which is subjectively made, and the second one has a problem relating to asset replications, which proves to be quite demanding on the market's side. The application of the two models can indeed be highly complicated, if not impossible, in a country like Tanzania.

Consider Dynamic Programming which comes with an exogenously determined discount rate. Tanzania is characterized by highly dynamic inflation. The only comfort here comes from the fact that the direction of the inflation is almost always known to be upward, and this may make inflation rate forecasts possible. But it should be noted that the frequency with which inflationary changes occur, can seriously affect precision in forecasts. On top of that, in order to reach an optimal solution, Dynamic Programming works backward in a recursive manner. Now consider the case of an infinite time horizon. Fundamentally, such a scenario frees the Bellman equation from the bondage of time t , but maintains it as a function of the state reached at that specific time. This will be true if the profit flow Pr , the discount rate ρ , and the transition probability distribution function are all

independent of time.

But while the above condition can be assumed in many economic applications, it's worth noting that in some parts of the world, especially where markets are highly fragmented and underdeveloped, such as Tanzania, the profit flow, discount rate, and the transition probability distribution are not totally independent of time. Therefore great care should always be taken in applying the method in different environments. Fortunately, however, cases of those variables being strongly dependent on time are almost unheard of, otherwise plenty of arbitrage opportunities would be created.

On the other side, Contingent Claims Analysis comes with the presumption that the risk in the price P , of the underlying asset could either be replicated or spanned by existing assets. When there's no replication, which is precisely the Tanzanian case, given the country's incomplete and highly fragmented markets¹⁵, the riskless portfolio cannot be constructed, hence we cannot generate a differential equation that would help us get the value of investment¹⁶, $V_p(P)$. The solution will therefore be to apply Dynamic Programming, which, as we said earlier, comes with an exogenously determined discount rate. Below, we enumerate a number of further problems which can directly be associated with the models' applicability in our parts of the world, and in particular within the Tanzanian economic scope.

THE GAMBLER'S CHOICE

We introduced above a special case in Dynamic Programming, when the choice in any given period is binary. As we saw above, this case involves the decision to either continue or terminate the project. The idea is simple: If the investment is made, a profit, $Pr(x)$ is generated, if no investment is made, a termination value $\Omega(x)$ is realized. Also notice that there'll be a capital gain or loss a period from now, which is the discounted expected value. Thus the optimal

value of the firm is expressed as:

$$(11) \quad F(x) = \max \left\{ \Omega(x), \Pr(x) + \frac{1}{1+\rho} E[F(x') | x] \right\},$$

where x is the state variable, ρ the discount rate, and $\Omega(x)$ is the termination value, depending on the state reached. Our interest here lies in the single cut-off point, mentioned above and denoted by x^* . Here we introduce what we call the Gambler's Choice. This idea derives from our experience of Tanzanian investment culture. A gambler may set a threshold, but as long as no wins are coming by the threshold may continuously be pushed beyond its initial value in anticipation of future big fortunes. Though this will usually depend on the gambler's financial base at the time, it can further be encouraged by borrowing, selling off, or mortgaging, owned assets.

The boundary conditions in the Contingent Claims approach are based on the idea that investors want to choose the option exercise date optimally to maximize the value of their assets. But we should not overlook the fact that when one is down and out there'll usually be an option to incur additional losses in the hope of getting a fortune. Trying to get a fortune, such investment strategies would usually lead to losing even the little that was available, and as we saw above, this usually leads to financial stress. We attribute this problem to subjective probability, without confining it to it.

THE REPLICATION OF RISK CHARACTERISTICS

Two more issues of great concern are associated with asset risk replication, and the assumption that the underlying uncertainty follows an Ito process¹⁷, both of which fall under Contingent Claims Analysis.

While Contingent Claims Analysis seems to ideally treat the discount rate, the

requirements under which this is achieved are just too demanding. Consider the requirement of replication, for instance, which simply means that the stochastic component of the return of the asset being replicated is required to be perfectly correlated with the replicating asset or portfolio of assets. The implication of being perfectly correlated is that it's not enough for the risk components to abide by the same probability rule, but they must also coincide in all realizations. Anyone can immediately realize that this can be quite hard, if not impossible, to achieve, even in highly developed markets, let alone those in the forgotten world.

To relax things a little bit, we can simply say that the applicability of Contingent Claims Analysis can only be thought of under highly developed markets with an extremely rich menu of traded risky assets. In other words, we exclude the best part of the world, including Tanzania, from the list of the beneficiaries of the method. This can be very disappointing, to say the least.

If we are to adopt the concept of the cutoff-line between stopping and continuing, we should then have a demarcating curve $x^*(t)$, between them. We can refer to this curve as the free boundary. Then the problem unfolds: How do we specify $x^*(t)$ alongside the value we seek to maximize, $F(x,t)$. We cannot seek the help of the theory of differential equations. It should be noted that the conditions applicable to free boundaries are application-specific and they must be governed by circumstantial considerations. Naively said, such a decision is discretionary, and as such create loopholes for irrational decisions. Most irrational decisions made by the Tanzanian government over the past three decades can be reflected in the foregoing consideration.

SOURCES OF INVESTIBLE FUNDS

There's another very serious problem associated with the applicability of Contingent Claims Analysis at a national level in Tanzania. The crippled economy of our country has developed a syndrome of not being able to sustain itself. This means that the best part of the government budget has to be funded from outside through loans, and occasionally, grants. Moral hazard problems have either been delaying the loans or holding them back altogether¹⁸. Needless to say, this leads to irregularities in both consumption and investment. Though investment conditions are gradually changing, in Tanzania, as is the case in other underdeveloped economies, the government is still the major investor. All public utilities are still state-owned, and they'll, probably, continue being so for quite some time.

It follows, therefore, that talking about optimal investment decisions at a national level, under such circumstances, seems to make very little sense, if at all it does. Here we are talking about waiting, temporary suspension, or total abandonment, being determined, not by the flow of information, but by the flow of loan-funds! Probably what we should initially focus on is an investigation into skipped options. Consider a situation when an option to invest is not exercised, not because one has opted for waiting for a more rewarding moment, given available information, but because, one simply had no means to invest!

Most least developed countries, and especially those in the Sub-Saharan region, Tanzania being one of them, depend much on foreign capital. This comes mainly in the form of loan capital, and it is therefore strongly related to the countries' foreign debt. The following figures roughly reveal the investment structure of most African countries¹⁹: The Sub-Saharan external debt shot up from US\$194.70bn in 1991 to US\$223.20bn in 1995, while external investment shot up from US\$22.60bn in 1991 to US\$60.00bn in 1995. Within the same period of time the figures for the entire African continent were US\$289.00bn in 1991 up to US\$314.00bn in 1995.

The figures reveal heavy dependency on foreign capital. This may not be a problem in its own right. The major problem is that the flow of these funds is highly unpredictable. It is the unpredictability of the flow of funds that leads to irrational investment decisions being made. The tug-of-war between Sub-Saharan Africa and what has come to be known as the donor community has been going on for years now, and it is likely to persist into the future. As a result of this, the Sub-Saharan region finds itself trapped between hope and despair, not knowing exactly when and how the funds will be made available, and as a result end up making arbitrary investment decisions.

RESULTS FROM WAITING

Most real options encountered in real life are not simple in structure. Think of a situation where you may want to bail out of a project at any given time, and the price at which you can bail out be as random as the time you do so. Further, reinstatement of the project will depend on the market performance. None of the three states can be known in advance. They'll all depend on probability distributions and forecasting methods which are likely to be inaccurate! The accuracy of modeling remains a function of accurate information, which cannot always be guaranteed. For instance, if you decide to wait, the implicit assumption is that by so doing you'll miss out on the first year's cash flow but you'll learn what would've been the cash flow had you gone ahead with the project. Sometime you learn virtually nothing! The estimate of the option value, however, depends much on what you learn as you wait.

COMPLETENESS OF RISK MARKETS

There's an issue of great importance that should be noted right from the beginning. This pertains to the general policy toward investment, and specifically to the completeness of risk markets. A look at Tanzania, over the past three decades or so, reveals that the government was, and to a greater extent still is, the leading investor. This trend was probably born of a political influence from eastern Europe and China, where centralized planning was once believed to combat both

investment risk and poor distribution of wealth! We can ignore this belief and simply say that the option to wait would still apply in such circumstances, as a good investment decision could be made even better by the flow of more information.

The immediate problem that a country like Tanzania faces, is that of incompleteness of its markets. In a complete risk market each firm will take the stochastic process of the price as given²⁰ and will have rational expectations about it. But things are different with the poorest economies of the world, Tanzania being one of them. Markets for risk are highly incomplete, as a result of which, a sort of justification for interventions is made. These are the so-called financial repressions, which have plagued the best part of Africa for decades. For a good number of years financial repressions have constituted a battle front between Sub-Saharan Africa and the Bretton Woods institutions.

Our experience tends to suggest that the disagreements between these two parties will probably never be resolved, temporary coercive solutions notwithstanding. Probably the Bretton Woods institutions have a good reason to be of the opinion they are. The most serious financial repressions include the setting of price ceilings and floors, urban rent controls and agricultural price supports. Dixit and Pindyck argue that "...price supports promote investment by reducing the downward risk. However, the resulting rightward shift of the industry supply function implies lower prices in good times.the overall result is a lower long-run average price.the policy is harming the very group it sets out to help"²¹. They further argue that these policies reduce overall economic efficiency, and admit that their finding is perhaps a politically more potent argument against them. This is very true, and, indeed, this is what has been the case over the years. Economic disputes between the two sides have been seriously politicized. This chain-reacts to further irrational decisions.

CONCLUSION

Our discussion of the two models revealed a strong symbiosis between them. Mathematically interpreted, the value function, which is the object to be manipulated under Dynamic Programming and the asset value, the replicating object under Contingent Claims Analysis are, in fact, solutions to very similar partial differential equations. Nevertheless, the discount rate, undoubtedly a vital component in the valuation of projects is treated differently by both methods. The discount rate ρ is exogenously specified as a part of the objective function, under Dynamic Programming. Such a practice is highly delicate and will usually require great care to handle.

On the other side, under Contingent Claims Analysis the required rate of return on an asset derives from the overall equilibrium in capital markets. Only the risk-free rate, r , is exogenous. However, the risk-free rate can be endogenized if the capital markets are assumed to be complete and in equilibrium. Thus it is evident that Contingent Claims Analysis treats the discount rate more realistically than does Dynamic Programming. Indeed, it does, but with a question mark. Yes, a question mark. As we saw above, Contingent Claims Analysis seems to ideally treat the discount rate, but the requirements under which this is achieved are just too demanding. We saw the requirement of replication, for instance, where the stochastic component of the return of the asset being replicated was required to be perfectly correlated with the replicating asset or portfolio of assets.

Reverting to Dynamic Programming, we see that though the method does not require too much, it is built on the foundations of subjective probability, which, as a rule, spells potential danger in decision-making²², and on the same grounds, testing the theory becomes very difficult if not impossible. However, given some reasonable and appropriate assumptions, the method can effectively be applied. Nevertheless, we should hasten to remind ourselves of our experiences of the past. Gigantic projects like the infamous Capital Development Authority in Dodoma once set in motion could never be

reversed, despite the enormous losses, both actual and potential, associated with them. The Treasury still runs the ill-fated Kunduchi Beach Hotel, in north-eastern Dar Es Salaam, until very recently, funds were still being pumped into the National Bank of Commerce in its death bed, and many, many more such cases. Apparently, all this reflects what we referred to as the gambler's choice.

COULD THE MODELS EVER BECOME APPLICABLE IN TANZANIA?

We noted above that the models' practicality depends much on their robustness. The above listed problems notwithstanding, if the assumptions hold, conditions are met and the requirements are fulfilled, there could be scope for applying the models. Here we should draw a line of demarcation between their potentials and applicability. To avoid the problem of elimination without substitution, at this level, we'll have to admit that the models could be workable, or at least potentially workable, and can be used in different controllable phenomena. If that is true, then the basic problem is associated with identifying, studying, understanding, as well as modeling them. If everything is in place, the models could find a wide range of applications, including:

- Capital Budgeting
- Allocation of Research and Development Funds, and
- Long-term Corporate Planning.

Under normal circumstances, and with a focus on progress, the Tanzanian investors, both current and potential, can reap the fruits of the techniques. Of course, we stress again that identifying their problems, thoroughly studying them, then modeling and validating them, will always be the primary problems to be solved first. What we set up in this paper is, undoubtedly, a continuation of what has existed for long, and as such we can not claim originality. Nevertheless, we believe it will keep on evolving in time and space, getting enhanced by different experiences at each level of its development. This is exactly what we seek to achieve. Our attention is basically focused on what we would like to call the Science of

Financial Decision-making, and looking at it in the context of the forgotten world. This perspective extends the horizons of the financial theory to cover various geographical locations and different economic scenarios. If anything, we would prefer to see our origination traced along that line.

NOTES

- ¹ Dixit, A. K. and Pindyck, R. S.; (1994).
- ² Dixit, A. K. and Pindyck, R. S.; (1994), page 16.
- ³ Brealey & Myers(1996), Principles of Corporate Finance..
- ⁴ Copeland and Weston(1988), Financial Theory and Corporate Policy.
- ⁵ Brealey & Myers(1996), Principles of Corporate Finance.
- ⁶ This situation is more pronounced in the underdeveloped world than it is in developed economies. Given the prevailing economic conditions in Tanzania, it will be ridiculous to even try to convince anyone that the interest rate offered by local banks does actually attract investors.
- ⁷ Dixit, A. K. and Pindyck, R. S; (1994), page 6-7.
- ⁸ Richard Bellman; (1957); Dynamic Programming.
- ⁹ R. E. Markland(1979); Topics in Management Science.
- ¹⁰ Since the Arusha Declaration in, 1967,

the Tanzanian Government has been the major investor in almost all sectors of the economy. Probably by adopting such an approach, the Government was laying down the foundations of all investment problems which were to follow.

See E. Bozorov; Sofia Press (1975); Applied Mathematics.

Consider the sum of the general geometric progression:

$$a + ar + ar^2 + \dots = S_n = \frac{a(1-r^{n+1})}{(1-r)} = \frac{a}{(1-r)} - \frac{ar^{n+1}}{(1-r)}$$

where a and r are the first term and common ratio, respectively. Now if r lies between zero and one, then

$r^n \rightarrow 0$ as $n \rightarrow \infty$, and since it cannot be negative, it must tend to some positive limit l . And since

$r^{n+1} = rr^n$, then both r^{n+1} and r^n , are ultimately equal to l , hence

we've $l = rl$, showing that, since r is not equal to unity, the limit l must be zero. The same applies when r lies between minus one and zero. Thus the value of the term

$$\frac{ar^n}{(1-r)} \rightarrow 0 \text{ as } n \rightarrow \infty, \text{ and the}$$

sum converges at $S_\infty = \frac{a}{1-r}$. And

notice that such a limit exists only for those geometric progressions with common ratios lying between -1 and 1 .

R. A. Howard(1971); Dynamic Probabilistic Systems; Vol. II: Semi-Markov and Decision Processes.

Tanzania has experienced quite a good number of such incidences in the past, with the most outstanding example being the Capital Development Authority project in Dodoma, and the most recent one, probably, being that of the National Bank of Commerce-currently restructuring into NBC(1997) Ltd and MicroFinance after 30 years of poor performance. Given the infrastructural and economic nature of the country, markets in Tanzania are likely to remain fragmented even after the inception of the Dar Es Salaam Stock Exchange, and apparently remain in that state for much longer afterwards.

The subscript in the symbol we use to denote the investment value, simply indicates a replicating portfolio of assets, and the bracketed P indicates that the investment value is a function of the price of the replicating asset or portfolio of assets.

When both the expected return, often referred to as the drift, and variance coefficients of a stochastic variable are a function of both the current state and time, the resulting continuous-time stochastic process $x(t)$, is called an Ito process.

Most Tanzanians can still vividly recall the loan-freezing saga in 1994/5, which led to the resignation of the late Professor Kighoma Malima, then Minister for Finance.

Source: Radio One; African Panorama; 1996. December. 22, 11:00-12:00hrs; Host: Charles Hillary. S. Lofthouse(1994); Readings in Investments.

Dixit, A. K. and Pindyck, R. S.(1994); page 19.

E. J. Dudewicz & S. N. Mishra(1988); Modern Mathematical Statistics.